Scalability Analysis of Exploration with Micro-Robots for Search in Rubble

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Abstract - To utilize a huge number of small robots, called *micro-robots*, for survivor searches in rubble in disaster areas is considered as a promising approach because of their smallness. How many micro-robots should be deployed is one of crucial research issues for survivor searches with micro-robots. In this paper, we derive theoretical lower bounds of the number of deployed micro-robots for accomplishing their search mission by modeling rubble as a graph and drawing orbits of micro-robots as paths on the modeled graph. As the first step to analyze difficulties in search in rubble with micro-robots, we focus on two relations: one is the relation between the number of deployed micro-robots and the sizes of rubble and the other is relation between the number of deployed micro-robots and abilities of their locomotion. Comparisons between the theoretical lower bounds and simulated results of the number of deployed micro-robots imply that searches in rubble with micro-robots becomes difficult as the rubble size in vertical directions becomes large.

Keywords: Search in Rubble, Micro-Robot, Graph-based Analysis

1 INTRODUCTION

As various robots have been developed [1]–[4], attempts to use robots for survivor searches in rubble in disaster areas are studied [5]. These robots can enter dangerous areas for humans and perform their tasks. Moreover, using robots may reduce risks of further endangering the survivors and rescuers due to secondary disasters. For these reasons, robots are expected as one of promising means for search and rescue in disaster areas. Since smallness of robots allows them to break into small gaps in rubble, it is more likely to find survivors buried inside the rubble. Therefore, to utilize smaller robots, which are often called *micro-robots* [6]–[8], for survivor searches in disaster areas is considered [9]–[11].

Searches in rubble with micro-robots are generally performed on the basis of deployment of enormous micro-robots because of the following two reasons: one is a simple structure of micro-robots and the other is a complex structure of rubble. Since it is difficult to install many functions on small micro-robots, they have minimum capabilities for searching, such as moving and detecting obstacles and survivors [9]. Thus, one approach for achieving a fast completion of the search is deploying enormous micro-robots and searching many possible spaces simultaneously [10]. As another point of view about the difficulty in searches in rubble with micro-robots, Cho and Arnold [11] pointed out a possibility that micro-robots may fall into holes surrounded with debris inside rubble and they cannot escape from the holes. Therefore, enormous micro-robots have to be deployed so that they accomplish the search in rubble even if a certain amount of them fall into such holes.

To determine how many micro-robots should be deployed to complete a search in rubble with micro-robots is one of crucial issues since the searches are performed with a huge number of micro-robots. However, few researches analyze the number of deployed micro-robots to complete the search in rubble. We simply refer to the number of deployed micro-robots to complete the search in rubble as the number of deployed micro-robots, hereafter. In [10], a method to reduce the number of deployed micro-robots is introduced but the target of the research is two-dimensional spaces with several obstacles. However, the number of deployed micro-robots will strongly depend on complexity of three-dimensional structures of rubble since many holes exist inside rubble of collapsed buildings, as Cho and Arnold pointed out in [11]. In this paper, we use the number of deployed micro-robots for indicating one of the difficulties in search in rubble with micro-robots and analyze the number of required micro-robots in a three-dimensional structure of rubble.

As the first step to understand difficulty in searches in rubble with micro-robots, we theoretically derive the *minimum number of deployed micro-robots* to complete the searches under an ideal case where all micro-robots move optimally in the rubble on the basis of the complete information about the internal structure of the rubble. We model a pile of rubble as a graph and get the minimum number of micro-robots by deriving the minimum number of paths, which correspond to orbits of the micro-robots, to cover all vertices in the modeled graph. Then, we conduct several simulation experiments assuming that currently developed micro-robots having capabilities of moving horizontally and detecting humans are used. By comparing the theoretical and simulation results, we discuss how searches in rubble with the currently developed micro-robots is difficult.

The rest of this paper is organized as follows. In Section 2, we define the problem of searches in rubble with micro-robots. Then, a method to model a pile of rubble with a graph and a method to derive the theoretical minimum number of deployed micro-robots are described in Section 3 and 4, respectively. We compare the number of deployed micro-robots in the case of the primitive micro-robots with the theoretical results in Section 5. Finally, we conclude this paper in Section 6.

2 PROBLEM FORMULATION OF SEARCH IN RUBBLE WITH MICRO-ROBOTS

In this section, we define the problem of searches in rubble with micro-robots.

2.1 Micro-robots and Searches in Rubble

This subsection describes micro-robots and searches in rubble.

1) Micro-robots: We assume that low-cost and small, cm-scale, micro-robots are used for searches in rubble in this paper. The micro-robot is equipped with 1) a sensor device to detect humans, 2) a moving mechanism to move inside rubble and 3) a device for notifying findings of survivors. Regarding the human detection in rubble, we assume that micro-robots use a temperature sensor or carbon a dioxide sensor to detect humans. Regarding the moving capability of micro-robots, we assume that they have the capability of moving in horizontal directions since many currently developed millimeter-scale micro-robots have the capability of crawling ground [6]–[8]. That is, we do not assume that micro-robots have the moving capability toward the upward direction in rubble. Finally, regarding the notifications of the survivor detection, we assume that micro-robots have wireless communication functionality like Wi-Fi or Bluetooth and they can notify the finds of survivors from anywhere inside the rubble to outside the rubble. Hence, we do not make further consideration about notifications of the survivor detection, hereafter.

2) Searches in Rubble: A micro-robot finds a survivor if and only if it detects the survivor by using the installed sensor device, i.e., the micro-robots need to approach close enough to the survivor. To find all survivors buried in the rubble, micro-robots need to arrive at all possible *spaces*, which are defined as places without any pieces of rubble, at least once, since no knowledge about locations of survivors is available in advance of the search.

Micro-robots have only the horizontal movement capability. Therefore, once they drop vertically into the lower space, they can never go back to the upper spaces. Since the micro-robots have no way to avoid dropping into *holes*, which are defined as spaces surrounded with walls of debris, micro-robots that drop into holes no longer continue their search missions. That is, it is practically impossible for one micro-robot to reach all spaces in rubble and find all survivors buried there. Hence, for search in rubble with micro-robots, it is necessary to deploy multiple micro-robots and to search all possible spaces simultaneously.

2.2 Building Artificial Rubble

In this section, we explain a method for building rubble to analyze the difficulty in searches in rubble with micro-robots. We refer to rubble built with our method as *artificial rubble*.

We model rubble by piling small and equal-sized cuboids, which are the minimum units of artificial rubble, in a three-dimensional Euclidean space. We divide the three dimensional Euclidean space into small sections. We refer to the three-dimensional Euclidean space and the sections there as *field* and *cells*, respectively. These cuboids represent small blocks of rubble. That is, the cuboids are building blocks of the artificial rubble and we build the complex structure of rubble by piling the cuboids. In this paper, we use a cuboid with sides of 10, 10, and 5 cm for constructing the artificial rubble. The sizes of cells and cuboids are the same. We construct artificial rubble by placing the cuboids into cells. We refer to cells filled with the cuboids, which represent objects of rubble, as *rubble cell* and other cells except for rubble cells as *empty cells*. That is, empty cells represent spaces inside rubble. We handle rubble as a field of given *width*, *depth*, and *height*. To simplify notations, we refer to the lengths in the x, y, and z axes directions in the Euclidean space as width, depth, and height, respectively. Micro-robots discussed above can stay in empty cells having a rubble cell underneath them. We refer to these cells, where micro-robots can stand as *plane cells*. They can move horizontally on plane cells that are adjacent each other on the same height of the field. We refer to a set of adjacent plane cells with the same height as a *plane*.

Figure 1 depicts the overview of constructing the artificial rubble. At the initial step, we fill a field of given width, depth, and height with empty cells. That is, the field is empty at the initial step. Then, we fill the field with rubble cells from the bottom toward the top step by step. At each layer of the field, rubble cells are placed until a given ratio, which we call *rubble ratio*, of cells are filled with rubble cells, as shown in Fig. 1(a). At each layer, one cell is selected randomly among empty cells. Then, we make a cuboid of $i \times j$ cells starting from the selected cell and fill with rubble cells in the cuboids. The integer numbers i and j are uniformly distributed random numbers between 1 and 10. Note that we make one constraint where at least one cells underneath the cuboid must be a rubble cell to avoid the situation where rubble cells float. We construct a complex structure by combining small rubble cells in this way, as shown in Fig. 1(b).

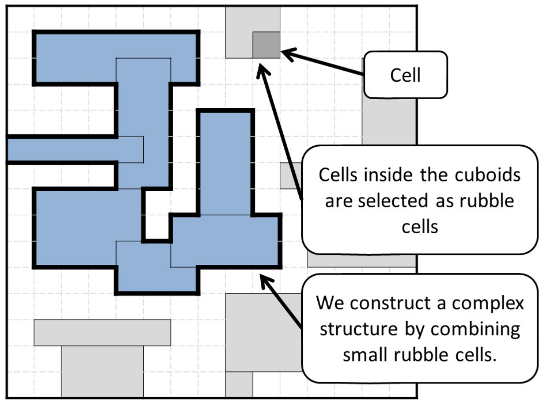
2.3 Definition of Searches in Rubble

This section defines the problem of searches in rubble with micro-robots. We define the completion of a search in rubble with micro-robots as the situation where all plane cells in artificial rubble are *covered* by micro-robots. A plane cell is covered when at least one micro-robot reaches the cell. Covering all plane cells means that micro-robots will find all survivors in the rubble. To cover all plane cells, a search in rubble with micro-robots are performed according to the following procedures. First, all micro-robots are deployed on top of the highest rubble cells. Each micro-robot moves independently from other micro-robots. Then, they move to one of adjacent empty cells of the same height in x or y axes directions per unit time. If the cell where micro-robots arrived at is an empty cell, they fall vertically to a plane cell right under the empty cell. Thus, if a plane is horizontally surrounded with rubble cells, they cannot escape from the plane. We refer to such planes as *holes*. We assume that micro-robots do not collide each other. Finally, we assume that micro-robots have enough battery capacity and therefore they do not stop moving till the completion of the search.

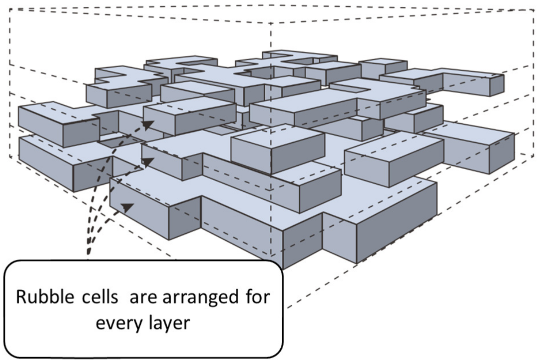
2.4 Difficulty in Searches in Rubble

This section explains a metric to indicate difficulty in a search in rubble with micro-robots and discusses a method for analyzing the relation between the size of rubble and the difficulty.

Since a micro-robot in a hole keeps staying there, all plane



(a) Decision of selection of rubble cells



(b) Example of artificial rubble

Figure 1: The overview of constructing artificial rubble

cells are not able to be covered by one micro-robot. Thus, we have to deploy multiple micro-robots to cover all plane cells. We expect that many micro-robots have to be deployed since many holes exist inside rubble of collapsed buildings. Therefore, the number of deployed micro-robots to complete the search is one of metrics to know how the search is difficult.

To measure the difficulty in the search in rubble with the number of deployed micro-robots, we compare results obtained by simulation with the theoretical lower bounds derived theoretically. In the simulation experiments, we assume that above-mentioned autonomous micro-robots are used, that is, micro-robots having only the horizontally moving capability move autonomously without any knowledge about rubble. We use the *minimum number of deployed micro-robots* to cover all plane cells as a metric to indicate the difficulty in the search in rubble. We simplify refer to the minimum number of deployed micro-robots as the minimum number of micro-robots, hereafter. That is, we use the minimum number of micro-robots as guidelines to measure the difficulty in the search.

To analyze the difficulty, we use a graph that represents a structure of the artificial rubble and derive the minimum number of micro-robots. We define the graph and a method to derive the minimum number of micro-robots from the graph in the following sections.

3 METHOD OF MODELING RUBBLE WITH A GRAPH

3.1 Overview

In this section, we define a search in rubble as a problem traversing a graph which corresponds to the artificial rubble constructed in the previous section. That is, we model the artificial rubble as a directed graph, of which vertices and directed edges correspond to places where micro-robots can be and paths where micro-robots can move, respectively. Then, we derive the minimum number of paths, which correspond to orbits of the micro-robots, to traverse all the vertices in the graph.

We model the artificial rubble as a directed graph according to the following two steps: 1) We simply express all possible places and paths where micro-robots can be and move as a directed graph and then 2) we simplify the graph by summarizing horizontal movements of micro-robots. More precisely, we first derive a graph G_1 by converting all empty cells, where micro-robots can be, as vertices and placing directed edges between all the possible vertices where micro-robots can move horizontally or fall vertically. That is, micro-robots can move from one vertex to another along directed edges in G_1. However, G_1 has many redundant information to know the minimum number of paths to cover all vertices, such as loops and unreachable vertices. Thus, we derive G_2 by summarizing several vertices in G_1 where micro-robots can move each other with horizontal movements to one vertex and removing unreachable vertices. That is, G_2 contains only directed edges that correspond to vertical movements of micro-robots. Consequently, the problem of a search in rubble is equivalent to covering all vertices in G_2 since all plane cell are contained in vertices in G_2 and the minimum number of micro-robots is equivalent to the minimum number of paths to cover all vertices in G_2. The following section mathematical defines the directed graphs and develops several heuristic algorithms to build the graphs.

3.2 Graph Formation

3.2.1 Formulating Rubble as a Graph

Before defining a directed graph, symbols used in this section are defined. The location of the cell in the artificial rubble is expressed by (i, j, k) $(i, j, k \in \mathbb{Z})$, which indicates i-th, j-th, and k-th cell in x, y, and z axis directions, respectively. The cell at (i, j, k) is represented as $c_{(i,j,k)}$. The function $C(i, j, k)$ is defined, which returns 0 if $c_{(i,j,k)}$ is an empty cell, return 1 if $c_{(i,j,k)}$ is a plane cell. X, Y, and Z are the width, depth, and height of the artificial rubble.

Let $G_1 = (V_1, E_1)$ be the directed graph, where V_1 and E_1 are the sets of vertices and edges. V_1 includes all plane cells in the artificial rubble and defined as follows:

$$
\begin{aligned}
V_1 = \{v_{(i,j,k)} \mid (i, j, k) \in I \times J \times K \ \wedge \ C(i, j, k) = 0 \\
\wedge \ (k = 1 \vee (k \neq 1 \wedge \ C(i, j, k-1) = 1)) \\
\vee \ (i, j, k) = (0, 0, Z+1)\}, \quad (1)
\end{aligned}
$$

where I, J, and K represent ranges of x, y, and z axes, respectively. That is, I is represented as $I = \{x \in \mathbb{N} \mid 1 \leq$

$x \leq X\}$ and J and K are represented similarly. The condition $C(i, j, k) = 0 \land (k = 1 \lor (k \neq 1 \land C(i, j, k-1) = 1)$ represents that $c_{(i,j,k)}$ is plane cell. Therefore, V_1 contains all the plane cells in the artificial rubble. Micro-robots are deployed on top of the rubble. Thus, we add another vertex to express the deployment of micro-robots, *root vertex* v_0, and the root vertex is represented by the constraint $(i, j, k) = (0, 0, Z + 1)$.

Next, we define the set of directed edges E_1 in G_1. Directed edges express the movement of micro-robots from one plane cell to another plane cell with horizontal movements and free falls and are defined as follows:

$$\begin{aligned} E_1 = \Big\{ \big(v_{(i,j,k)}, v_{(l,m,n)}\big) \mid \\ & (l, m, n) = (i \pm 1, j, k) \ \lor (l, m, n) = (i, j \pm 1, k) \\ & \lor \ (((l = i \pm 1 \ \land \ m = j) \ \lor \ (l = i \ \land \ m = j \pm 1)) \\ & \qquad \land \ \textstyle\sum_{h=n}^{k} C(l, m, h) = 0 \ \land \ k > n) \\ & \lor \left((i, j, k) = (0, 0, Z+1) \land \sum_{h=n}^{Z} C(l, m, h) = 0 \right) \Big\}. \end{aligned} \tag{2}$$

The condition $(l, m, n) = (i \pm 1, j, k)$ and $(l, m, n) = (i, j \pm 1, k)$ represents the horizontal movements and $(((l = i \pm 1 \ \land \ m = j) \ \lor \ (l = i \ \land \ m = j \pm 1)) \ \land \ \sum_{h=n}^{k} C(l, m, h) = 0 \ \land \ k > n)$ represents movements from one plane cell to another in a free fall through empty cells. To express the deployment of micro-robots, E_1 contains the edge that is link from the root vertex v_0 to vertices of plane cells of the top of the rubble and this is represented by the last condition in Eq. (2).

Next, we present a heuristic algorithm to derive the directed graph G_1 from given artificial rubble in Algorithm 1. First, the sets of vertices and edges V_1 and E_1 are initialized with empty sets at lines 1 and 2. From line 3 to 11, G_2 is constructed from the given artificial rubble according to the definitions in Eqs. (1) and (2). The sub-function SET_EDGE defined at lines 12 to 15 is the function to set edges between vertices.

3.2.2 Summarizing a Graph

Next, we derive the graph $G_2 = (V_2, E_2)$ by summarizing several vertices in G_1 where micro-robots can move each other with horizontal movements to one vertex. That is, vertices in V_2 correspond to planes in the artificial rubble. A plane is denoted by $A_{(i,j,k)}$, which contains the plane cell $c_{(i,j,k)}$, and the constraint to aggregate plane cells and constructing a plane is expressed as

$$A_{(i,j,k)} = \{c_{(i,j,k)} \mid c_{(i,j,k)} \in A_{(i,j,k)s} \ \land \ A_{(i,j,k)s} = A_{(i,j,k)s-1}\}, \tag{3}$$

where $A_{(i,j,k)\,s}$ is defined using the following constraints.

$$A_{(i,j,k)\,1} = \{c_{(i,j,k)}\} \tag{4}$$

$$\begin{aligned} A_{(i,j,k)\,s} = \{c_{(l,m,n)} \mid & \ c_{(l,m,n)} \in A_{(i,j,k)\,s-1} \\ & \lor \ (c_{(l,m,n)} \in Q \\ & \land \ \exists c_{(s,t,u)} \in A_{(i,j,k)\,s-1} \ ((l, m, n) = (s \pm 1, t, u) \\ & \lor \ (l, m, n) = (s, t \pm 1, u)))\} \end{aligned} \tag{5}$$

$$\begin{aligned} Q = \{c_{(i,j,k)} \mid & \ (i, j, k) \in I \times J \times K \ \land \ C(i, j, k) = 0 \\ & \land \ (k = 1 \lor (k \neq 1 \land \ C(i, j, k-1) = 1))\} \end{aligned} \tag{6}$$

Algorithm 1 Constructing $G_1 = (V_1, E_1)$

Input: The artificial rubble, $c_{(i,j,k)}$
Output: The directed graph, $G_1 = (V_1, E_1)$

1: $V_1 \leftarrow \emptyset$
2: $E_1 \leftarrow \emptyset$
3: **for** $k \leftarrow 1$ to Z **do**
4:   **for** $j \leftarrow 1$ to Y **do**
5:     **for** $i \leftarrow 1$ to X **do**
6:       **if** $C(i, j, k) = 0 \land C(i, j, k-1) = 1$ **then**
7:         $V_1 \leftarrow V_1 \cup \{v_{(i,j,k)}\}$
8:       **end if**
9:     **end for**
10:   **end for**
11: **end for**
12: **for all** $v_{(i,j,k)} \in V_1$ **do**
13:   SET_EDGE$(v_{(i,j,k)}, c_{(i\pm1,j,k)})$
14:   SET_EDGE$(v_{(i,j,k)}, c_{(i,j\pm1,k)})$
15: **end for**

16: **function** SET_EDGE$(v_{(i,j,k)}, c_{(l,m,n)})$
17:   **if** $C(l, m, n) = 0$ **then**
18:     **if** $n = 1 \ \lor \ C(l, m, n-1) = 1$ **then**
19:       $E_1 \leftarrow E_1 \cup \{(v_{(i,j,k)}, v_{(l,m,n)})\}$
20:     **else**
21:       $z \leftarrow n$
22:       **while** $C(l, m, z) = 0$ **do**
23:         $z \leftarrow z - 1$
24:       **end while**
25:       $E_1 \leftarrow E_1 \cup \{(v_{(i,j,k)}, v_{(l,m,z+1)})\}$
26:     **end if**
27:   **end if**
28: **end function**

To express the relationship between a plane and a vertex v_a that constitutes the plane a, we introduce S_{v_a}, where S_{v_a} is a set of plane cells that constitute the plane a. S_{v_a} satisfies $\forall c_{(i,j,k)} \in S(v_a) \forall c_{(l,m,n)} \in S(v_a) \ (A_{(i,j,k)} \in A_{(l,m,n)})$. S_{v_0} is an empty set.

Next, we explain how a set of directed edges E_2 of directed graph G_2 is constructed. Edges in E_2 represent the vertical movements in free falls from one plane to the next plane. The edge from v_a to v_b must satisfy $\exists \, c_{(i,j,k)} \in S(v_a) \ \exists \, c_{(l,m,n)} \in S(v_b) \, (((l = i \pm 1 \ \land \ m = j) \ \lor \ (l = i \ \land \ m = j \pm 1)) \ \land \ k > n)$. Since micro-robots are deployed via the root vertex v_0, we connect the root vertex to all other planes which can reach from the top of the artificial rubble, i.e., directed edges are placed from v_0 to all v_a that satisfies the constraint $\exists \, v'_{(i,j,k)} \in S(v_a) \ \sum_{h=n}^{Z} C(l, m, h) = 0$.

Then, we present a heuristic algorithm to derive the directed graph G_2 from the given rubble and the directed graph G_1 built with Algorithm 1. In The algorithm consists of three parts: initializing variables from lines 1 to 3, constructing vertices from lines 4 to 14, and constructing edges from lines 15 to 25. To distinguish which plane vertices belong to, we assign identifiers to all plane and the identifier of plane where $c_{(i,j,k)}$ belongs to is stored to $c_{(i,j,k)}.id$. The initial value of $c_{(i,j,k)}.id$ is zero. From lines 4 to 14, each plane in the given artificial rubble is converted to a vertex. That is, all adjacent

Algorithm 2 Constructing $G_2 = (V_2, E_2)$

Input: $G_1 = (V_1, E_1)$ and $c_{(i,j,k)}$
Output: $G_2 = (V_2, E_2)$
1: $V_2 \leftarrow \emptyset$
2: $E_2 \leftarrow \emptyset$
3: $id \leftarrow 1$
4: **for** $k \leftarrow 1, Z$ **do**
5:   **for** $j \leftarrow 1, Y$ **do**
6:     **for** $i \leftarrow 1, X$ **do**
7:       **if** $C(i, j, k) = 0 \wedge C(i, j, k-1) = 1$
8: a $\wedge c_{(i,j,k)}.id = 0$ **then**
9:         $V_2 \leftarrow V_2 \cup \{v_{id}\}$
10:         CLUSTER$(id, c_{(i,j,k)})$
11:         id $\leftarrow id + 1$
12:       **end if**
13:     **end for**
14:   **end for**
15: **end for**
16: **for** $k \leftarrow 1, Z$ **do**
17:   **for** $j \leftarrow 1, Y$ **do**
18:     **for** $i \leftarrow 1, X$ **do**
19:       **if** $C(i, j, k) = 0 \wedge C(i, j, k-1) = 1$ **then**
20:         $id \leftarrow c_{(i,j,k)}.id$
21:         SET_EDGE2$(v_{id}, c_{(i\pm1,j,k)})$
22:         SET_EDGE2$(v_{id}, c_{(i,j\pm1,k)})$
23:       **end if**
24:     **end for**
25:   **end for**
26: **end for**

plane cells of the same height are aggregated toe one plane. To construct planes, we use sub-function CLUSTER, which aggregates plane cells recursively assign the identifier to the plane cells. From lines 15 to 25, directed edges are placed from one plane to another where micro-robots can move vertically in free falls. Finally, we construct G'_2 by removing vertices that are unreachable from v_0 and edges to the removed vertices since micro-robots cannot reach such spaces in the rubble.

4 THE MINIMUM NUMBER OF MICRO-ROBOTS

The theoretical minimum number of micro-robots is equivalent to the number of paths in the minimum path cover of the modeled graph G'_2. A path cover is a set of directed paths such that every vertex in the graph belongs to at least one of the path. If the path cover consists of the minimum number of paths, the path cover is referred to as the minimum path cover. In this section, we first explain the reason why the minimum number of micro-robots is equivalent to the number of paths in the minimum path cover and then how to derive the minimum path cover of G'_2.

A search in the rubble with micro-robots is equivalent to traverses of vertices leaving from the root vertex along with directed edges on the directed graph derived from the rubble. Specifically, we have modeled a given pile of artificial rubble as a directed graph. Passing through a vertex in the graph is equivalent to surveying the space in the rubble that correspond

Algorithm 3 Sub-functions for constructing $G_2 = (V_2, E_2)$

function SET_EDGE2$(v_{id}, c_{(i,j,k)})$
  $z \leftarrow k$
  if $C(i, j, z) = 0 \;\wedge\; C(i, j, z-1) = 0$ **then**
    while $C(i, j, z) = 0$ **do**
      $z \leftarrow z - 1$
    end while
    $id2 \leftarrow c_{(i,j,k)}.id$
    $E_2 \leftarrow E_2 \cup \{(v_{id}, v_{id2})\}$
  end if
end function

function CLUSTER$(id, c_{(i,j,k)})$
  if $C(i, j, k) = 0 \wedge C(i, j, k-1) = 1 \wedge c_{(i,j,k)}.id = 0$ **then**
    $c_{(i,j,k)}.id \leftarrow id$
    CLUSTER$(id, i \pm 1, j, k)$
    CLUSTER$(id, i, j \pm 1, k)$
  end if
end function

to the vertex. In the similar way, moving along with a directed edge is equivalent to moving from one space to another in the rubble. Therefore, orbits of micro-robots in the rubble can be expressed as paths starting from the root vertex v_0 in the directed graph G'_2. The minimum number of deployed micro-robots is equivalent to the minimum number of paths, which are originating from v_0, to cover all vertices in G'_2.

Next, we derive the minimum number of paths originating from the root vertex v_0 for covering all the vertices in G'_2. A path P is defined as an ordered set of vertices. We denote a set of paths using $\mathcal{P}$, hereafter. The set of paths to cover all vertices is also called as *path cover* and it is defined as follows: $\mathcal{P}$ is a path cover of $G = (V, E)$ if $\mathcal{P}$ is a set of paths of G such that every $v \in V$ is included at least one path $P \in \mathcal{P}$ [12]. That is, a path cover must satisfy the following condition:

$$\forall v \in V \;\; \exists P \in \mathcal{P} \;\; v \in P. \tag{7}$$

The minimum path cover is a path cover $\mathcal{P}$ such that $|\mathcal{P}|$ is minimum.

The orbits of micro-robots can be expressed by paths but the paths must start with v_0 since micro-robots are deployed on top of the rubble. That is, the paths of micro-robots must satisfy the following constraint:

$$\forall P \in \mathcal{P} \;\; (v_0 \in \mathcal{P}) \;\wedge\; \forall\, v \in V \;\; \exists P \in \mathcal{P} \;\; (v \in P) \tag{8}$$

Such a path cover is called a single starting point path cover. Therefore, the constraints of paths of the path cover problem and the search in rubble with micro-robots are slightly different, as shown in Eqs. (7) and (8). The minimum single starting point path cover is a single starting path cover $\mathcal{P}$ such that $|\mathcal{P}|$ is minimum. Though algorithms to deriving the minimum path cover have been already developed but no algorithms to derive the minimum single starting point path cover.

If the number of elements of the minimum single starting point path cover is equal to that of the minimum path cover, the minimum number of micro-robots can be derived by solving the problem of the minimum path cover of G'_2. We prove that the number of elements of the minimum single starting point path cover is equal to that of the minimum path cover as follows: Since for all vertices $v \in V_2'$ in G_2' at least one path that originates from v_0 and reaches to v exists, a set of paths $\mathcal{P}$ exists such that $\mathcal{P}$ satisfies the following condition, $|C| = |\mathcal{P}| \wedge \forall P \in \mathcal{P}\ (v_0 \in P) \wedge \forall P' \in C\ \exists P \in \mathcal{P}\ (P' \subseteq P)$, where C is the minimum path cover of G_2'. That is, the set of paths $\mathcal{P}$ is the minimum path cover and the all paths in $\mathcal{P}$ start with v_0. The number of elements of the minimum path cover is equal to that of the minimum single starting point path cover. Hence, the minimum number of micro-robots can be derived by solving the problem of the number of elements of the minimum path cover of G_2'.

Finally, we explain a method to derive the minimum path cover in G_2'. G_2' is a directed acyclic graph (DAG) [12]. It is a well-known fact that the minimum vertex-disjoint path cover in DAG can be derived by solving the maximum matching problem by converting the DAG into a bipartite graph [13]. The minimum vertex-disjoint path cover is a set of the minimum number of elements in vertex-disjoint path covers. The vertex-disjoint path cover is a set of paths $\mathcal{P}$ such that for every $v \in V$ in G there exists at exactly one path $P \in \mathcal{P}$ including v. The vertex-disjoint path cover must satisfy another constraint that no paths in the set cannot share vertices in addition to the minimum path cover. However, if the target graph G is a DAG, the minimum path cover of G is equal to the minimum vertex-disjoint path cover of the transitive closure of the graph G_{clo} [12]. Therefore, we can have the minimum path cover of G_2' by deriving the minimum vertex-disjoint path cover of transitive closure of G_2'. In this way, the minimum number of micro-robots deployed to complete the search in the rubble can be derived.

5 DIFFICULTY IN SEARCH IN RUBBLE WITH MICRO-ROBOTS

In this section, we analyze the difficulty in searches rubble with micro-robots. First, we describe our simulation environments. Then, we analyze the number of deployed micro-robots and the minimum number of deployed micro-robots by change the width/depth and height of the artificial rubble.

5.1 Simulation Conditions

We set parameters of the artificial rubble as follows: To express complicated shape of rubble, we set the size of one cell to $10 \times 10 \times 5$ cm. Since we suppose that micro-robots search inside highly dense rubble, where a large robot cannot enter, we set the percentage of rubble cells in the artificial rubble to reasonably high, i.e., 0.65. In our simulation experiments, micro-robots are deployed randomly on top of the artificial rubble. In the rubble, the micro-robots move independently to each other according to the Lévy Flight mobility model [14], [15], which is know as an efficient mobility pattern to search for targets. Lévy Flight is a random walk where the step lengths during the walk are described by a heavy-tailed probability distribution. In the simulation experiments, the Lévy distribution is used to describe the step lengths of the walk. The micro-robots move horizontally on planes and they fall in free falls when they reach an empty cell. In this paper, we ignore situations where micro-robots stop working during the search due to several issues, such as failures of locomotion or sensor devices. We define that a plane cell is surveyed if at least one micro-robot enters the plane cell. The search will be finished within 24 hours since the probability of humans under rubble being alive decreases rapidly 24 hours after they are buried. We compute the number of deployed micro-robots to cover 90% of plane cells in the artificial rubble within 24 hours. We compute the average of results obtained from 30 simulation trials. In the following section, we investigate how the search in rubble is difficult by evaluating the number of deployed micro-robots by changing the size of the artificial rubble.

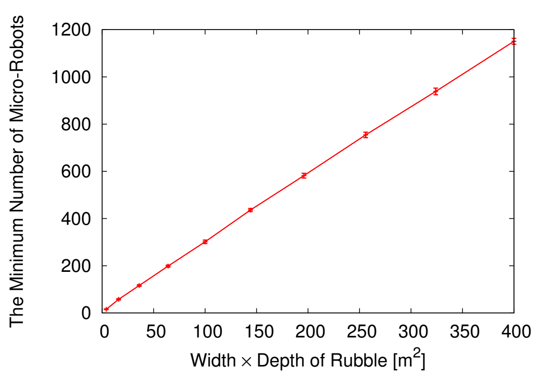
5.2 The Number of Deployed Micro-robots

First, we observe the effects of the area size, i.e., the width multiplied by the depth, of the artificial rubble on the number of deployed micro-robots. Figure 2 shows relations between the number of deployed micro-robots and the area size of the rubble. In this simulation, the height of the rubble is set to 1.5 meters. The horizontal axes of the figures are the area size, which is defined as the width multiplied by the depth. The minimum number of micro-robots derived theoretically is shown in Fig. 2(a) and the number of deployed micro-robots obtained through simulations is shown in Fig. 2(b). The error bars in Fig. 2(b) indicate the 95% confidence intervals. Note that comparing the absolute values of the analytical and simulation results is nonsense. We compare the tendency of the results in this paper. Both the minimum number of micro-robots derived theoretically and the number of deployed micro-robots obtained through simulations increase almost proportionally to the area size.

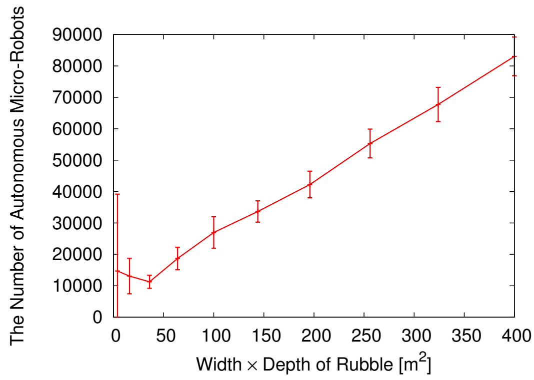
Next, we observe the effects of the height of the artificial rubble on the number of deployed micro-robots in Fig. 3. The horizontal axes indicate the height of the artificial rubble. The vertical axes are the same as those in Fig. 2. The minimum number of micro-robots derived theoretically is shown in Fig. 3(a) and the number of deployed micro-robots obtained through simulations is shown in Fig. 3(b). In this simulation, both the width and depth of the artificial rubble is set to 10 meters. In contrast to the minimum number of micro-robots derived theoretically, which increases almost proportionally to the height, the number of deployed micro-robots obtained through simulations increases more sharply.

5.3 Effects of Locomotion in Vertical Directions

One of the critical reasons why such huge number of micro-robots must be deployed for accomplishing searches in rubble is that micro-robots may fall into holes surrounded with debris inside rubble and they cannot escape from the holes, as Cho and Arnold have pointed out in [11]. One solution to resolve this issue is installing an ability of locomotion in vertical directions, such as jumping locomotion [16], on micro-robots. This section investigates how an ability of locomotion in vertical directions relaxes the difficulty in searches in rubble with

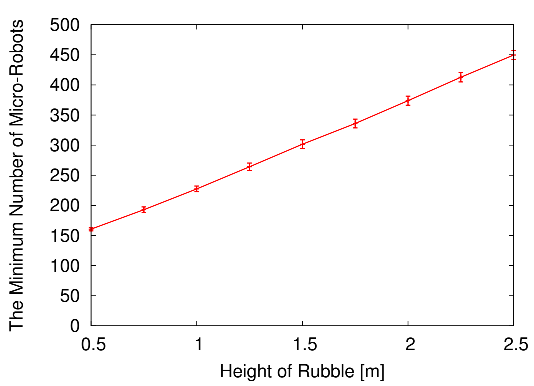


(a) The minimum number of micro-robots derived theoretically

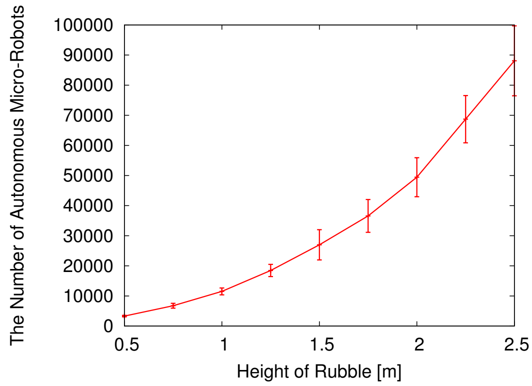


(b) The number of deployed micro-robots derived by simulations

Figure 2: The minimum number of micro-robots and the number of deployed micro-robots in the case that the area size of the rubble is changed



(a) The minimum number of micro-robots derived theoretically



(b) The number of deployed micro-robots derived by simulations

Figure 3: The minimum number of micro-robots and the number of deployed micro-robots in the case that the height of the rubble is changed

micro-robots.

As an ability of locomotion in vertical directions, we introduce *jumping locomotion*, which is proposed in several studies [9], [16]. Let us assume that micro-robots can jump h m high. That is, they can go to cells in h m higher places. We refer to the height of jumping locomotion as *jumping height*, in this section. We use the following settings regarding the behavior of micro-robots: when micro-robots arrive at a wall, where they cannot move without the jumping locomotion, they jump with the probability of 0.1; otherwise they continue to move in the opposite direction with the horizontal locomotion.

In this simulation, both the width and depth of the artificial rubble is set to 3 meters, which is smaller than the settings of simulations for Fig. 3, to focus on the effect of the jumping locomotion. Regarding other simulation settings, we use the same ones as those used in the previous section. Figure 4 shows the number of deployed micro-robots in the case that the height of the rubble is changed. To investigate effects of the jumping locomotion, we compare the number of deployed micro-robots without the jumping locomotion and that with jumping locomotion. We set the jumping height, h, to 0.05 m, in this simulation. Installing the jumping locomotion drastically reduces the number of deployed micro-robots. To investigate effects of the jumping height, we evaluate the number of deployed micro-robots with changing the jumping height, and the results are shown in Fig. 5. The results indicate that increasing the jumping height does not impact on the reduction in the number of deployed micro-robots. Finally, we compare the number of deployed micro-robots with the jumping locomotion derived via simulations (Fig. 6) with the minimum number of micro-robots derived theoretically (Fig. 3(a)). In contrast to the theoretical minimum number of micro-robots, which increases almost proportionally to the height, the number of deployed micro-robots with the jumping locomotion increases more sharply in the same way as micro-robots without the jumping locomotion (Fig. 3(b)).

5.4 Implications

Observations found through the evaluations are summarized as follows. The number of deployed micro-robots is mostly proportional to the size in the horizontal direction of each rubble, as shown in Fig. 2. In contrast, it steeply increases as the size in the vertical direction of rubble increases, as shown in Fig. 3. Installing the jumping locomotion function

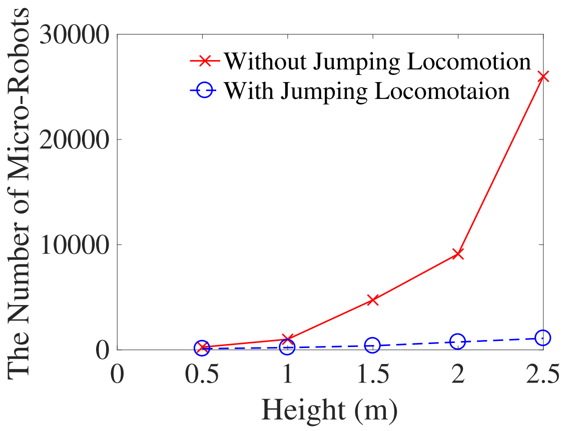


Figure 4: The number of deployed micro-robots in the case of micro-robots with and without the jumping locomotion

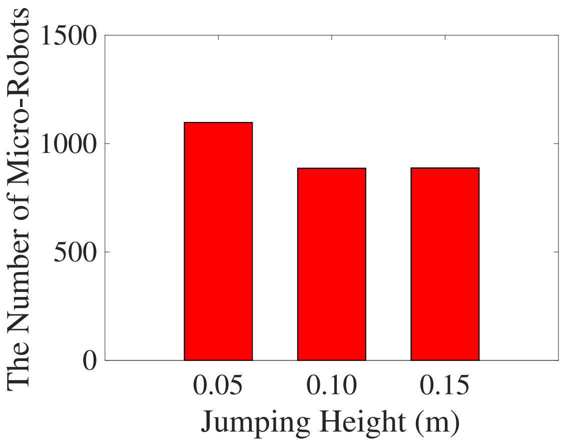


Figure 5: The effects of the jumping height on the number of deployed micro-robots

is a candidate to relax the difficulty in the searches in rubble with micro-robots, as shown in Fig. 4, which shows that the jumping locomotion function drastically reduces the number of deployed micro-robots. However, the results also indicate that the jumping locomotion function does not resolve the root cause of the difficulty fundamentally due to the fact that the number of deployed micro-robots with the jumping locomotion increases more sharply than the theoretical minimum number of micro-robots, which is almost proportional to the height of rubble.

Through simulation experiments, we have learned the following implications: First, searches in rubble with primitive micro-robots which have only the horizontal movement function as their locomotion function are very difficult since many micro-robots are required to accomplish the searches. Second, the search in rubble using micro-robots with the jumping locomotion function might be useful for searches in rubble if it is not very high. Nevertheless, using such micro-robots may not be a feasible solution in the case that the rubble is high, such as high buildings. Finally, we can roughly estimate the number of deployed micro-robots to accomplish a search in rubble of an ordinary dwelling house in Japan, using the observation found through the simulation, i.e., the number of deployed micro-robots is mostly proportional to the size in the horizontal

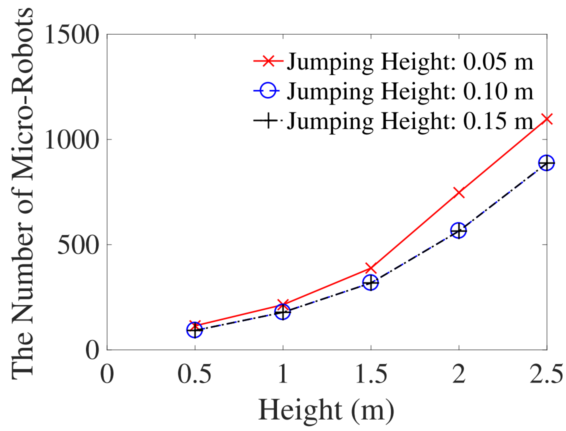


Figure 6: The number of deployed micro-robots in the case of micro-robots with the jumping locomotion

direction of each rubble, as shown in Fig. 2. Assuming that the average total floor size per a dwelling in Japan is 94.42 m^2 in 2013 [17] and the height of the rubble is about 2.5 m, the number of deployed micro-robots is roughly estimated at $9298.3 = 886.3 \times 94.42/9.0$ according to the result that 886.3 micro-robots are required for the search for 9 $m^2 \times 2.5$ m cuboid, as shown in Fig. 6. This estimation implies that micro-robots with the jumping locomotion function can be used for searches rubble of low dwelling houses in Japan.

6 CONCLUSION

In this paper, we analyze the number of deployed micro-robots to complete the search in the rubble. To investigate the difficulty in the search, we derive the theoretical lower bounds of the number of deployed micro-robots, which can be used as guideline to measure the difficulty in the search in the rubble. As the first step to analyze the difficulty in the search in the rubble, we compare the number of deployed primitive micro-robots, which have only the horizontal movement function as their locomotion function, with the minimum number of micro-robots derived theoretically by changing the area size and the height of the rubble. In contrast to the theoretical minimum number of micro-robots, which increases almost proportionally to the height, the number of deployed micro-robots obtained through simulations increases more sharply. Although installing the jumping locomotion to micro-robots drastically reduces the number of deployed micro-robots, the number of deployed micro-robots with the jumping locomotion also increases more sharply than the theoretical minimum number of micro-robots. This fact suggests that the jumping locomotion may not essentially resolve the root cause of the difficulty in searches in rubble, and hence searches in rubble with primitive micro-robots get difficult as the rubble gets large in vertical directions.

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