A Design Method of Optimal H_2 Integral Servo Controller for Torsional Vibration System

Noriyuki Komine*, Masakatsu Nishigaki*, Kunihiro Yamada** and Tadanori Mizuno***

* Graduate School of Science and Technology, Shizuoka University, Japan
** Department of System Design, Tokyo Metropolitan University, Japan
*** Faculty of Information Science, Aichi Institute of Technology, Japan
komine@keyaki.cc.u-tokai.ac.jp, nisigaki@inf.shizuoka.ac.jp
yamada@kunighiroi.com, mizuno@mizulab.net

Abstract -This paper proposes a design method of optimal H_2 integral servo controller. The optimal H_2 integral controller is to establish a way to find the admissible controller such that the controlled plant is stabilized and guarantee to track a constant reference signal while minimizing the close-loop system H_2 norm from the disturbance to the controlled output. In this paper the derivative state constrained optimal H_2 integral servo controller is proposed for oscillatory system with a constant disturbance. This design method has the advantage that it can be applied to reduce the vibration of the two-inertia system. The effectiveness of proposed controller is evaluated by experiments.

Keywords: Optimal controller, Integral servo, Torsional vibration

1 INTRODUCTION

In sense of optimal control, the state feedback approaches for a linear dynamical system which not only stabilizes but also dampens the output responses of closed-loop system is generally required [1]. It is also required that the output of a system has no steady-state error for a desired input even if the parameter variations or disturbances exist. Consequently, the integral servo problem was initiated by H. W. Smith and E. J. Davison [2], in which they proposed the state and output integral feedback approaches by the differential state transformations, and gave the feedback control which contains a feed forward based on the measurable disturbance by the affine transformation. In addition, the optimal regulator control theory was primarily proposed by R.E. Kalman [3] to minimize the quadratic performance index of state variables and inputs. By using the regulator theory, the design method of an optimal tracking system by introducing the integral action for the system was obtained and reported by T. Takeda and T. Kitamori [4]. However, it is difficult to select the proper values of the weighting matrices of performance index in the optimal servo problem to mitigate under damped responses of dynamic systems. Besides, the optimal H_2 servo problem is to find the optimal control such that the output tracks the desired trajectory, minimizing the tracking error cost and state excitation cost in the sense of an optimal H_2 control [5]-[6]. On the other hand, Anderson and Moore [7] introduced an optimal controller with a prescribed degree of stability affecting the locations of all closed-loop poles. However, it does not necessarily reduce the under damping of the closed-loop system. Recently, the optimal H_2 control for oscillatory system minimizing a performance criterion involved time derivatives of state vector was formulated to mitigate the vibration responses of dynamic systems [9]-[13].

In this paper, the theorem of the derivative state constrained optimal H_2 integral servo controller is obtained by the standard H_2 control framework [8]. The proposed controller is applied to reduce the vibration of the two-inertia system. The verification of the effectiveness of the proposed controller to reduce the vibration responses and to reject the constant disturbance is shown by experiments.

2 H_2 INTEGRAL SERVO PROBLEM

In order to obtain the optimal H_2 integral servo controller, the following controlled plant equations are given as

$$\begin{aligned} &\frac{d}{dt}x(t) = Ax(t) + B_2u(t) + d_0,\ x(0) = x_0, \\ &y(t) = C_2x(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^{m2}$, $d_0 \in R^1$ and $y(t) \in R^{p2}$ denote the state vector, the input vector, the constant disturbance and the output vector, respectively. The integral $x_I(t) \in R^{p2}$ of the error vector $e(t) \in R^{p2}$ between the reference input $r(t) \in R^{p2}$ and controlled output $y(t)$ is defined as

$$x_I(t) = \int_0^t e(\tau)d\tau,\ e(t) = r(t) - y(t). \tag{2}$$

Using Eq. (1) and Eq. (2), the augmented controlled plant is then given by

$$\begin{aligned} &\frac{d}{dt}\begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix}u(t) + \begin{bmatrix} d_0 \\ r(t) \end{bmatrix}, \\ &y(t) = \begin{bmatrix} C_2 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix}. \end{aligned} \tag{3}$$

In order to the steady state tracking error $\lim_{t \to \infty} e(t)$ should be vanished, the derivative augmented state vector defined as $\dot{x}_I(t)$ which should be vanished for approaching $t \to \infty$. The derivative augmented system is given by combining of the derivative state equation of Eq. (3) with design parameter matrices as

$$P(s): \begin{cases} \dfrac{d}{dt}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \left[\begin{array}{cc:cc:cc} B_1 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & B_{1I} & 0 & 0 & 0 & 0 \end{array}\right]\dot{w}(t) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix}\dot{u}(t) \\ \dot{z}(t) = \left[\begin{array}{cc} C_1 & 0 \\ 0 & C_{1I} \\ \hdashline 0 & 0 \\ 0 & 0 \\ \hdashline 0 & 0 \end{array}\right]\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \hdashline D_{11} & 0 \\ 0 & D_{11I} \\ \hdashline 0 & 0 \end{array}\right]\begin{bmatrix} \ddot{x}(t) \\ \ddot{x}_I(t) \end{bmatrix} + \left[\begin{array}{c} 0 \\ 0 \\ \hdashline 0 \\ 0 \\ \hdashline D_{12} \end{array}\right]\dot{u}(t)) \\ \begin{bmatrix} \dot{y}(t) \\ y_I(t) \end{bmatrix} = \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \left[\begin{array}{cc:cc:cc} 0 & 0 & 0 & 0 & D_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{21I} \end{array}\right]\dot{w}(t), \end{cases} \tag{4}$$

where $B_1, B_{1I}, C_1, C_{1I}, D_{11}, D_{11I}, D_{12}, D_{21}$ and D_{21I} are denoted the design parameter matrices. $y_I(t) \in R^{p2}$ is added to obtain the proposed integral servo controller.

The disturbance $\dot{w}(t) = \left[\dot{w}_1^T(t) \quad \dot{w}_2^T(t) \;\vdots\; \ddot{x}^T(t) \quad \ddot{x}_I^T(t) \;\vdots\; \dot{w}_3^T(t) \quad \dot{w}_4^T(t)\right]^T$ is continuously differentiable in time. By definition of the optimal H_2 integral servo problem, the augmented general plant is given by

$$\hat{P}(s): \begin{cases} \dfrac{d}{dt}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \breve{B}_1\dot{w}(t) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix}\dot{u}(t) \\ \dot{z}(t) = \breve{C}_1\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \breve{D}_{12}\dot{u}(t)) \\ \begin{bmatrix} \dot{y}(t) \\ \dot{y}_I(t) \end{bmatrix} = \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \breve{D}_{21}\dot{w}(t), \end{cases} \tag{5}$$

where

$$\breve{B}_1 = \left[\begin{array}{cc:cc:cc} B_1 & 0 & AD_{11} & 0 & 0 & 0 \\ 0 & B_{1I} & -C_2D_{11I} & 0 & 0 & 0 \end{array}\right],$$

$$\breve{D}_{21} = \left[\begin{array}{cc:cc:cc} 0 & 0 & C_2D_{11} & 0 & D_{21} & 0 \\ 0 & 0 & 0 & D_{11I} & 0 & D_{21I} \end{array}\right],$$

$$\breve{C}_1 = \left[\begin{array}{cc} C_1 & 0 \\ 0 & C_{1I} \\ \hdashline D_{11}A & 0 \\ -D_{1I}C_2 & 0 \\ \hdashline 0 & 0 \end{array}\right], \breve{D}_{12} = \left[\begin{array}{c} 0 \\ 0 \\ \hdashline D_{11}B_2 \\ 0 \\ \hdashline D_{12} \end{array}\right].$$

Statement of Derivative State Constrained H_2 integral servo problem:

Let $r(t)$ denote the step reference vector. Derivative State Constrained Optimal H_2 servo integral problem is to find an admissible optimal integral controller such that the controlled plants with augmented integrator is stabilized and the output $y(t)$ tracks the constant reference signal $r(t)$ while minimizing the H_2 norm of the closed-loop transfer function with controlled plant from $L[\dot{w}(t)]$ to $L[\dot{z}(t)]$ of $\hat{P}(s)$.

3 SOLUTION OF STATMENT

The solution to the derivative state constrained H_2 optimal control defined above is given by the following procedure:

(i) Variable Linear transformation for the prescribed degree of stability [7].

(ii) Singular value decomposition and variable transformation to obtain the standard H_2 structure.

(iii) Hamilton matrix for obtaining the stabilizing solution of the H_2 optimal control problem.

3.1 Variable transformations

In order to consider the effect of the prescribed degree of stability α to a controller, each vector variable is exponentially weighted as follows.

$$\begin{bmatrix} \breve{\dot{x}}(t) \\ \breve{\dot{x}}_I(t) \end{bmatrix} = e^{\alpha t}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix}, \tag{6}$$

$$\begin{aligned} \breve{\dot{w}}(t) &= e^{\alpha t}\dot{w}(t), \\ \breve{\dot{z}}(t) &= e^{\alpha t}\dot{z}(t), \\ \breve{\dot{u}}(t) &= e^{\alpha t}\dot{u}(t), \end{aligned} \tag{7}$$

$$\begin{bmatrix} \breve{\dot{y}}(t) \\ \breve{\dot{y}}_I(t) \end{bmatrix} = e^{\alpha t}\begin{bmatrix} \dot{y}(t) \\ \dot{y}_I(t) \end{bmatrix}. \tag{8}$$

Hence, the generalized plant $P_\alpha(s)$: shown in Eq. (9) after applying the transformed vector variables Eq. (6)-Eq. (8) is given by

$$P_\alpha(s): \begin{cases} \dfrac{d}{dt}\begin{bmatrix} \breve{\dot{x}}(t) \\ \breve{\dot{x}}_I(t) \end{bmatrix} = \left[\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I\right]\begin{bmatrix} \breve{\dot{x}}(t) \\ \breve{\dot{x}}_I(t) \end{bmatrix} + \breve{B}_1\breve{\dot{w}}(t) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix}\breve{\dot{u}}(t) \\ \breve{\dot{z}}(t) = \breve{C}_1\begin{bmatrix} \breve{\dot{x}}(t) \\ \breve{\dot{x}}_I(t) \end{bmatrix} + \breve{D}_{12}\breve{\dot{u}}(t)) \\ \begin{bmatrix} \breve{\dot{y}}(t) \\ \breve{\dot{y}}_I(t) \end{bmatrix} = \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \breve{\dot{x}}(t) \\ \breve{\dot{x}}_I(t) \end{bmatrix} + \breve{D}_{21}\breve{\dot{w}}(t), \end{cases} \tag{9}$$

The solution to the derivative state constrained H_2 optimal control defined above is given by procedure (ii) and (iii).

3.2 Singular value decomposition

There always exist unitary matrices $V_j, U_j, j = 1,2$ for the singular value decomposition of $\breve{D}_{12}$ and $\breve{D}_{21}$;

$$\breve{D}_{12} = U_1 \begin{bmatrix} 0 \\ \Sigma_1 \end{bmatrix} V_1,\ \Sigma_1 = \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ & & \sigma_{1m_2} \end{bmatrix},\ m_2 = \dim[\breve{\dot{u}}(t)],$$

$$\breve{D}_{21} = U_2 [0 \quad \Sigma_2] V_{12},\ \Sigma_2 = \begin{bmatrix} \sigma_{11} & & \\ & \ddots & \\ & & \sigma_{1p_2+p_2} \end{bmatrix},\ p_2 + p_2 = \dim \begin{bmatrix} \breve{\dot{y}}(t) \\ \breve{y}_I(t) \end{bmatrix},$$

where Σ_1, Σ_2 are the diagonal singular value matrices. Using the results obtained above, the input and output vectors as well as the generalized plant are accordingly transformed as shown in the following.

The generalized plant can be obtained by using the following variable transformations defined by

$$\breve{\dot{w}}(t) = V_2 \dot{w}(t), \tag{10}$$

$$\dot{z}(t) = U_1^T \breve{\dot{z}}(t), \tag{11}$$

$$\breve{\dot{u}}(t) = V_1 \Sigma_1^{-1} \dot{u}(t), \tag{12}$$

$$\begin{bmatrix} \dot{y}(t) \\ \dot{\breve{y}}_I(t) \end{bmatrix} = \Sigma_2^{-1} U_2^T \begin{bmatrix} \dot{y}(t) \\ y_I(t)(t) \end{bmatrix}. \tag{13}$$

Substituting Eq. (10)-Eq. (13) into Eq. (9), then the transformed generalized plant $P_\alpha(s)$: which is reduced to the standard form of the H_2 control problem is then obtained as

$$\hat{P}_\alpha(s): \begin{cases} \dfrac{d}{dt}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} = \left[\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I\right]\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{B}_1 \dot{w}(t) + \hat{B}_2 \dot{u}(t) \\ \dot{z}(t) = \hat{C}_1 \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{D}_{12} \dot{u}(t) \\ \begin{bmatrix} \dot{y}(t) \\ \dot{y}_I(t) \end{bmatrix} = \hat{C}_2 \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} + \hat{D}_{21} \dot{w}(t), \end{cases} \tag{14}$$

where

$$\hat{B}_1 = \breve{B}_1 V_2,$$

$$\hat{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix} V_1 \Sigma_1^{-1},$$

$$\hat{C}_1 = U_1^T \breve{C}_1,$$

$$\hat{C}_2 = \Sigma_2^{-1} U_2^T \begin{bmatrix} C_2 & 0 \\ 0 & I \end{bmatrix},$$

$$\hat{D}_{12} = U_1^T \breve{D}_{12} V_1 \Sigma_1^{-1} = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$\hat{D}_{21} = \Sigma_2^{-1} U_2^T \breve{D}_{21} V_2 = [0 \quad I].$$

Suppose that the transformed generalized plant $\hat{P}_\alpha(s)$ of Eq. (14) satisfy the following relations:

(A1) $\left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I,\ \hat{B}_2,\ \hat{C}_2\right)$ is stabilizable and detectable.

(A2) $\hat{D}_{12}$ and $\hat{D}_{21}$ have full rank.

(A3) $\begin{pmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I - j\omega I & \hat{B}_2 \\ \hat{C}_1 & \hat{D}_{12} \end{pmatrix}$ has full column rank for all ω.

(A4) $\begin{pmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I - j\omega I & \hat{B}_1 \\ \hat{C}_2 & \hat{D}_{21} \end{pmatrix}$ has full row rank for all ω.

The first assumption (A1) is for the stabilizability of the transformed generalized plant (14). The assumption (A2) is sufficient to ensure the controller is proper. The assumption (A3) and (A4) guarantee two Hamiltonian matrices belong to dom(Ric).

3.3 Hamiltonian matrices

Under the above assumptions (A1)-(A4), the optimal H_2 solution to the transformed generalized plant (14) is given as follows;

A couple of Hamiltonian matrices are constituted as

$$H_2 = \begin{bmatrix} \left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I\right) - \hat{B}_2 \hat{D}_{12}{}^T \hat{C}_1 & -\hat{B}_2 \hat{B}_2{}^T \\ -\hat{C}_1{}^T \hat{C}_1 + \hat{C}_1{}^T \hat{D}_{12} \hat{D}_{12}{}^T \hat{C}_1 & -\left(\left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I\right) - \hat{B}_2 \hat{D}_{12}{}^T \hat{C}\right)^T \end{bmatrix}, \tag{15}$$

$$J_2 = \begin{bmatrix} \left(\left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^T + \alpha I\right) - \hat{B}_1 \hat{D}_{21}^T \hat{C}_2\right)^T & -\hat{C}_2{}^T \hat{C}_2 \\ -\hat{B}_1 \hat{B}_1{}^T + \hat{B}_1 \hat{D}_{21}^T \hat{D}_{21} \hat{B}_1{}^T & -\left(\left(\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^T + \alpha I\right) - \hat{B}_1 \hat{D}_{21}^T \hat{C}_2\right) \end{bmatrix}. \tag{16}$$

Then, it is guaranteed that the solutions exist, which make $\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I + \hat{B}_2 \hat{F}_2$ and $\begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \alpha I + \hat{L}_2 \hat{C}_2$ stable.

From the couple of Riccati solutions,

$$X_2 = Ric(H_2) > 0,\ Y_2 = Ric(J_2) > 0, \tag{17}$$

it is able to construct the following optimal solution

$$\hat{K}_{H_{2\alpha}}(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \hat{B}_2 \hat{F}_2 + \hat{L}_2 \hat{C}_2 & -\hat{L}_2 \\ \hline \hat{F}_2 & 0 \end{array}\right] \tag{18}$$

to the transformed generalized plant (14), where

$$\hat{F}_2 = -(\hat{B}^T{}_2 X_2 + \hat{D}_{12}\hat{C}_1),$$
$$\hat{L}_2 = -(Y_2\hat{C}_2^T + \hat{B}_1\hat{D}_{21}.$$

The optimal control is then

$$\dot{\hat{u}}(t) = \hat{F}_2\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{\hat{x}}_I(t)\end{bmatrix}, \hat{F}_2 = \begin{bmatrix}\hat{F}_{2x} & \hat{F}_{2x_I}\end{bmatrix}. \quad (19)$$

A general control formulation with the derivative state constrained optimal H_2 integral servo controller $\hat{K}_{H_{2\alpha}}$ is given by the general configuration shown in Fig. 1. Consequently, the assumptions supposed above (A1), (A2), (A3) and (A4) can be reduced to the following expressions.

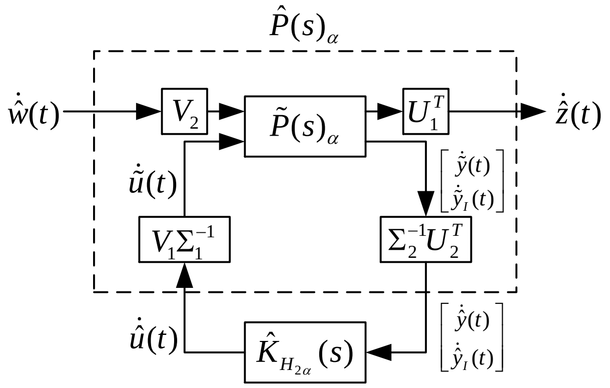

Figure 1: Block diagram of the structure for closed-loop system with equation (18).

Lemma: Suppose the system parameter matrix in equation (14) satisfy the assumptions (A1), (A2), (A3) and (A4), then following assumptions hold;

(A1)' $\left(\begin{bmatrix}A & 0\\ -C_2 & 0\end{bmatrix}, \begin{bmatrix}B_2\\ 0\end{bmatrix}, \begin{bmatrix}C_2 & 0\\ 0 & I\end{bmatrix}\right)$ is stabilizable and detectable.

(A2)' D_{12} and $\begin{bmatrix}D_{21} & 0\\ 0 & D_{21I}\end{bmatrix}$ have full rank.

(A3)' $\begin{pmatrix}\begin{bmatrix}A & 0\\ -C_2 & 0\end{bmatrix} - j\omega I & \begin{bmatrix}B_2\\ 0\end{bmatrix}\\ \begin{bmatrix}C_1 & 0\\ 0 & C_{1I}\end{bmatrix} & D_{12}\end{pmatrix}$ has full column rank for all ω.

(A4)' $\begin{pmatrix}\begin{bmatrix}A & 0\\ -C_2 & 0\end{bmatrix} - j\omega I & \begin{bmatrix}B_1 & 0\\ 0 & B_{1I}\end{bmatrix}\\ \begin{bmatrix}C_2 & 0\\ 0 & I\end{bmatrix} & \begin{bmatrix}D_{21} & 0\\ 0 & D_{21I}\end{bmatrix}\end{pmatrix}$ has full row rank for all ω.

Proof of Lemma: It is clearly shown that the optimal solution for the transformed generalized plant (14) can be obtained under the assumptions (A1)'-(A4)'by applying the facts of the rank properties (A1)-(A4) [13].

4 MAIN RESULT

By integrating the transformed generalized plant (14) with respect to time t with all initial values equal to zero, the optimal servo controller is obtained by following theorem. Thus, the optimal H_2 servo control solution for the system (14) is given by Eq. (18) of the theorem under the assumptions (A1)', (A2)', (A3)' and (A4)'. We have the following main result.

Theorem (Derivative State Constrained Optimal H_2 Integral Servo controller)

The derivative state constrained H_2 integral servo controller for the controlled plant (5) is given as

$$K_{H_{2\alpha}}(s) = \left[\begin{array}{c|c}\begin{bmatrix}A & 0\\ -C_2 & 0\end{bmatrix} + \begin{bmatrix}B_2\\ 0\end{bmatrix}F_{2\alpha} + L_{2\alpha}\begin{bmatrix}C_2 & 0\\ 0 & I\end{bmatrix} & -L_{2\alpha}\\ \hline F_{2\alpha} & 0\end{array}\right]$$

(20)

under the assumptions (A1)', (A2)', (A3)' and (A4)', where

$$F_{2\alpha} = V_1\Sigma_1{}^{-1}F_2 = -V_1\Sigma_1{}^{-1}V_1{}^T\left\{\begin{bmatrix}B_2^T & 0\end{bmatrix}X_2 + D_{12}{}^T\breve{C}_1\right\},$$

$$L_{2\alpha} = L_2\Sigma_2{}^{-1}U_2{}^T = -\left\{Y_2\begin{bmatrix}C_2^T & 0\\ 0 & I\end{bmatrix} + \breve{B}_1\breve{D}_{21}{}^T\right\}U_2\Sigma_2{}^{-1}U_{.2}^T\cdot$$

The optimal control is then

$$\dot{u}(t) = F_{2\alpha}\begin{bmatrix}\dot{x}(t)\\ \dot{x}_I(t)\end{bmatrix}, F_{2\alpha} = \begin{bmatrix}-F_{2\alpha x} & F_{2\alpha x_I}\end{bmatrix}. \quad (21)$$

Using Eq. (21), the optimal control law with integral feedback is written by

$$u(t) = -F_{2\alpha x}x(t) + F_{2\alpha x_I}\int_0^t e(t)dt\,. \quad (22)$$

This theorem can be proved as follow.

Proof: As the facts of the rank properties of the Lemma, this immediately shows that the optimal solution (20) for the generalized plant (5) implies the theorem under the assumptions of (A1)', (A2)', (A3)' and (A4)'. This concludes the proof of the theorem. □

5 OPIMAL FEEDBACK CONTROL AND TRACKING ERROR

For using the feedback control (21), the optimal control allows tracking of constant reference input in the infinity of time.

$$\begin{aligned} \dot{u}(t) &= -F_{2\alpha x}\dot{x}(t) + F_{2\alpha x_I}\dot{x}_I(t) \\ &= \begin{bmatrix} -F_{2\alpha x} & F_{2\alpha x_I} \end{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t) \end{bmatrix} + F_{2\alpha x_I} r(t) \end{aligned} \tag{23}$$

Then from Eq. (1) and Eq. (23), we have

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \end{bmatrix} &= \begin{bmatrix} I & 0 \\ -F_{2\alpha x} & F_{2\alpha x_I} \end{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ F_{2\alpha x_I} \end{bmatrix} r(t) + \begin{bmatrix} d_0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -F_{2\alpha x} d_0 \end{bmatrix}. \end{aligned} \tag{24}$$

This of Eq. (24) approach zero as $t \to \infty$, then the all steady-state variables are constant which is given by

$$\begin{aligned} \begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ -F_{2\alpha x_I}^{-1} F_{2\alpha x} & F_{2\alpha x_I}^{-1} \end{bmatrix} \cdot \\ &\left(\begin{bmatrix} 0 \\ F_{2\alpha x_I} \end{bmatrix} r(t) + \begin{bmatrix} d_0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -F_{2\alpha x} d_0 \end{bmatrix} \right). \end{aligned} \tag{25}$$

This of Eq. (25) approach zero as $t \to \infty$, then the output is tracked the reference input r(t) and does not effective the disturbance as follow:

$$\begin{aligned} y(t) &= \begin{bmatrix} C_2 & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} \\ &= \begin{bmatrix} C_2 & 0 \end{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ -F_{2\alpha x_I}^{-1} F_{2\alpha x} & F_{2\alpha x_I}^{-1} \end{bmatrix} \cdot \\ &\left(\begin{bmatrix} 0 \\ F_{2\alpha x_I} \end{bmatrix} r(t) + \begin{bmatrix} d_0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -F_{2\alpha x} d_0 \end{bmatrix} \right) \\ &= \begin{bmatrix} C_2 & 0 \end{bmatrix} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) = r(t). \end{aligned} \tag{26}$$

Then, the optimal control allows that the output is tracking of constant reference input in the infinity of time while rejecting the constant disturbance of Eq. (1).

6 EXPERIMENTAL RESULTS

A torsional vibration is occurred to the speed of motor by connecting flexible shaft. The vibration is an impediment to improve the characteristics of the two-inertia system. The experimental results of the speed control of the two-inertia system using the proposed controller will be shown to be effective to suppressing the vibration in this section. A structure of two-inertia system is shown in Fig. 2.

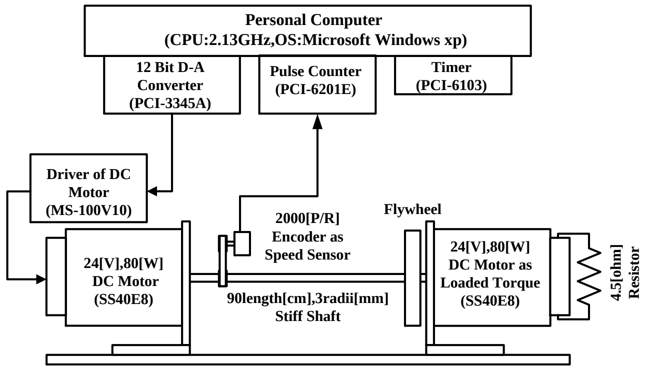

Figure 2: Structure of the two-inertia system.

In this system, two motors are connected by long shift (90cm) with the spring constant of the shift. The left side of motor is driving the right side of the load motor with the shift. By using Newton's second law, the linear dynamic equation of the two-inertia system with constant disturbance T_L is represented by

$$\begin{aligned} J_m \frac{d\omega_m}{dt} + F_m\omega_m &= u(t) + \tau_d, \\ J_L \frac{d\omega_L}{dt} + F_L\omega_L &= \tau_d - T_L, \\ \frac{d\tau_d}{dt} &= K_s(\omega_m - \omega_L), \end{aligned} \tag{27}$$

where J_m, J_L, F_m, F_L and K_s are the inertia of motor, the inertia of load, the friction of motor, friction of load and spring constant of the shaft, respectively. For tracking reference input, the integral $x_I(t)$ of the error vector $e(t)$ between the reference input $r(t)$ and controlled output $\omega_m(t)$ is defined as

$$x_I(t) = \int_0^t e(\tau)d\tau,\ e(t) = r(t) - \omega_m(t). \tag{28}$$

The parameters of the augmented controlled plant (3) is given by

$$A = \begin{bmatrix} F_m/J_m & 0 & 1/J_m & 0 \\ 0 & -F_L/J_L & 1/J_L & 0 \\ K_s & -K_s & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1/J_m \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$d_{0=} = -T_L,\ C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}.$$

The state variables of Eq. (3) is given by, $x(t) = [\omega_m(t)\ \ \omega_L(t)\ \ \tau_d(t)\ \ x_I(t)]^T$, where $\omega_m(t)$ denotes the speed of motor at time t, $\omega_L(t)$ denotes the speed of load at time t, $\tau_d(t)$ represents the torque of shaft and $\omega_m(t)$ denotes the disturbance torque. The numerical values of J_m, J_L, K_s are shown in Table 1. In the case of the numerical values, the friction of motor, friction of load and spring constant of the shaft are neglected, respectively.

Table 1: Numerical values of two-inertia system.

$J_m[Kg \cdot m^2]$	$J_L[Kg \cdot m^2]$	$K_s[N/m]$
0.0866	0.0866	400

The designing parameters $B_1, B_{1I}, C_1, C_{1I}, D_{11}, D_{11I}, D_{12}, D_{21}$ and D_{21I} in the generalized plant of Eq. (4) are chosen as:

$$\left.\begin{array}{c} C_1 = B_1^T = diag[\sqrt{10^{qi}} \quad \sqrt{10^{qi}} \quad \sqrt{10^{qi}}] \\ C_{1I} = B_{1I}^T = [\sqrt{20000}] \\ D_{11} = diag[\sqrt{e^{ni}} \quad \sqrt{e^{ni}} \quad \sqrt{e^{ni}}] \\ D_{11I} = \left[\sqrt{100}\right] \\ D_{12} = [1] \\ \begin{bmatrix} D_{21} & \\ & D_{21I} \end{bmatrix} = diag[\sqrt{0.01} \quad \sqrt{0.01}] \end{array}\right\}, \qquad (29)$$

where qi is the standard weighting parameter of C_1 and B_1 and ni is the weighting parameter of D_{11} for reducing the vibration.

By using the theorem of the main result, the feedback control laws for ni= -12 and ni= -5.5 are given by Eq. (30) and Eq. (31), respectively.

For ni= -12, the control law is obtained as:

$$\dot{u}(t) = -\left[5.321 \quad 1.903 \quad 0.014 \;\vdots\; -141.366\right]\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix}. \qquad (30)$$

For ni= -5.5, the control law is obtained as:

$$\dot{u}(t) = -\left[20.805 \quad -12.299 \quad 1.305 \;\vdots\; -123.289\right]\begin{bmatrix} \dot{x}(t) \\ \dot{x}_I(t) \end{bmatrix}. \qquad (31)$$

Regarding $\alpha = 20$ as the prescribed degree of stability, the variation of closed-loop poles when the parameter ni vary from -9 to -2 is shown in Fig. 3. It is seen that the original poles of the open-loop system locate on the imaginary axis. It verifies that the pair of poles with imaginary part approach to the real axis when the parameter ni becomes large. It seems that the vibration of speed is reduced by design parameter ni.

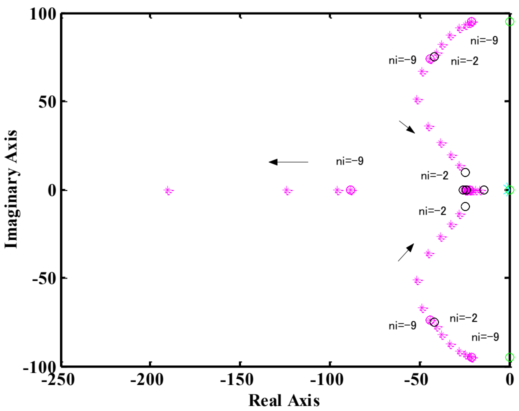

Figure 3: Closed-loop poles location for varying from ni= -9 to ni= -2 and $\alpha = 20$.

In experimental results, it is shown that the effectiveness of the controller can be reduced the vibration by the parameter ni. The experimental results in shown in Fig. 4-Fig. 6. In Figure 4, the oscillatory response occurred for selecting the weak design parameter ni= -12 of Eq. (32). However, the oscillatory response can be reduced for selecting the design parameter ni= -5.5.

$$D_{11} = diag\left[\sqrt{e^{ni}} \quad \sqrt{e^{ni}} \quad \sqrt{e^{ni}} \quad \sqrt{20000}\right] \qquad (32)$$

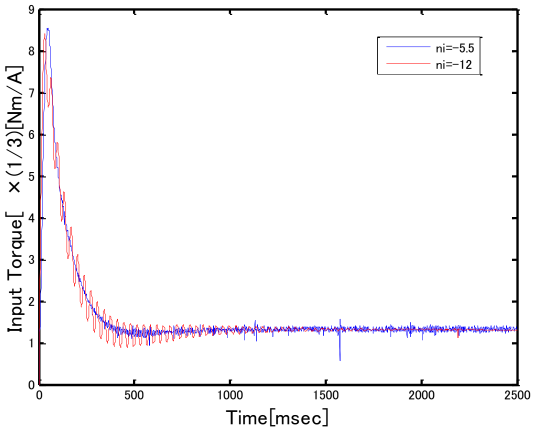

Figure 4: Responses of input torque of motor for ni= -12 and ni= -5.5.

Figure 5 shows the close loop responses of this plant with the feedback control gain of Eq. (30) and Eq. (31) for setting the reference speed 2500[RPM], respectively. Significantly, the output speed of motor is tracking the reference speed with removed the torsional resonance by the designing parameter ni= -5.5.

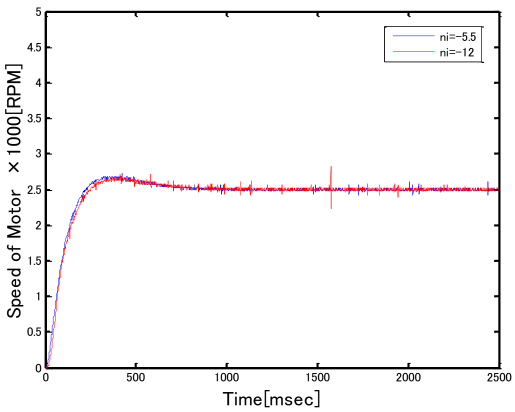

Figure 5: Responses of speed of motor for setting reference speed 2500[RPM] when ni= -12 and ni= -5.5.

In order to confirm the rejection of disturbance, Figure. 6 show that the response of speed is recovered the steady state when the load disturbance torque $T_L[N \cdot m]$ is applied to the motor after driving steady state speed of motor.

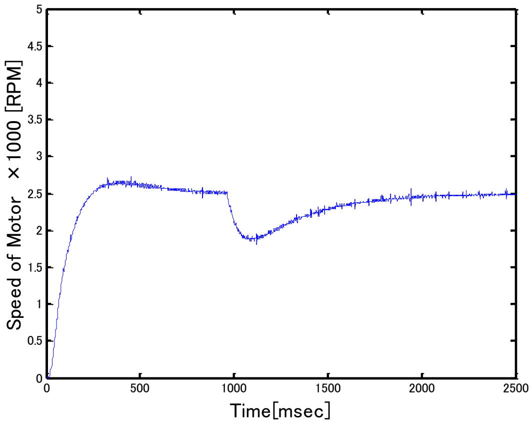

Figure 6: Response of speed of motor for applying the disturbance $T_L = 0.142[N \cdot m]$.

7 CONCLUSION

The optimal H_2 controller using derivative state constrained optimal H_2 integral servo controller has been proposed. The proposed controller is effective to reduce the vibration responses of the controlled system by H_2 control framework. It is recognized that the design parameter ni of matrix D_{11} can applied to the oscillation system with the reference inputs as well as constant disturbance. The experimental results have verified that the proposed schemes can be effective to reduce the oscillation and to mitigate the effect of the constant disturbance for the two-inertia system. The optimal H_2 controller with derivative state constraint will provide a method for improving vibration by comparing with other optimal control methods for future research.

REFERENCES

[1] J. J. Hench, C. He, V. Kucera and Meheman, “Damping Controllers via Riccati Equation Approach,” IEEE Tran. On AC, Vol. 43, No. 9, pp. 1280-1284 (1998).

[2] H. W. Smith and E. J. Davison, “Design of Industrial Regulators,” Proc. IEE, Vol. 119, No. 8, pp. 1210-1216 (1972).

[3] R. E. Kalman, “When Is a Linear Control System Optimal?” ASMEJ. Basic Engineering, ser. D, Vol. 86, pp. 51-60 (1964).

[4] T. Takeda and T. Kitamori, “A Design Method of Linear Multi-Input-Output Optimal Tracking Systems,” Trans. SICE, Vol. 14, No. 4, pp. 359-364 (1978).

[5] M. Ikeda and N. Suda, “Synthesis of Optimal Servosystems,” Trans. SICE, Vol. 24, No. 1, pp. 40-46 (1988).

[6] I. Masubuchi, A. Ohara and N. Suda, “A Design of Robust Servo Systems for Structured Uncertain Plants,” Proc. SICE, Vol. 30, No. 9, pp. 1051-1059 (1994).

[7] B. D. O Anderson and J. B. Moore, “Linear System Optimization with Prescribed Degree of Stability,” Proc. IEE, Vol. 116, No. 12, pp. 2083-2089 (1969).

[8] J.C.Doyle, K.Glover, P.P.Khargonrkar and B.A. Fran cis, “State-space solution to standard H_2 and H contr ol problem,” IEEE Trans. Automatic Control, Vol. A C-34, pp. 831-847 (1988).

[9] T. Trisuwannawat, K. Tirasesth, M. Iida, N. Komine and Y. Ochiai, “Derivative State Constrained Optimal H_2 Control for Oscillatory Systems and Its Application,” Trans. IEEJ (Industry Applications Society; IAS), Vol. 120-D, No. 6, pp. 775-781 (2000).

[10] N. Komine and K. Yamada, “Optimal H_2 Integral Co ntroller Design with Derivative State Constraints for Torsional Vibration Model,” Proc. KES2010, pp. 545 -554 (2010).

[11] W. Ichiyama, N.Komine, T. Benjanarasuth and M. Y oshida, “Optimal H_2 Integral Servo Controller with Derivative State Constraints for Suppressing under Damping of Oscillatory System,” Proc. JSST2011, pp.436-440 (2011).

[12] N. Komine, M. Nishigaki, T. Mizuno and K. Yamada, “A Design Method of Derivative State Constrained H_2 Integral Servo Controller for Suppressing under Damping of Oscillatory System,” Proc. AsiaSim201 2, Part I, pp. 515-162 (2012).

[13] N. Komine, M. Nishigaki, K. Yamada and T. Mizuno, “A Design Method of Optimal H_2 Integral Servo Pro blem,” Proc. IWIN2013, pp.141-148 (2013).

(Received March 11, 2014)
(Revised May 7, 2014)

Noriyuki Komine received the B. Eng. and M. Eng. degree in Electrical Engineering from the School of Engineering, Tokai University in 1972 and 1974, respectively. Since 1988, he has been an Associate Professor at the Department of Control Engineering and the Dept. of Embedded Technology of Tokai University. He is a member of IEEJ, SICE and the Japan Society for Simulation Technology. His research interests include optimal control and robust control for oscillation system.

Masakatsu NISHIGAKI received his Ph.D. in Engineering from Shizuoka University, Japan. He served as a Postdoctoral Research Fellow of the Japan Society for the Promotion of Science in 1995. Since 1996 he has been engaged in research at the Faculty of Informatics, Shizuoka University. He is now a Professor at the Graduate

School of Informatics of Shizuoka University. His research interests are in wide variety of information security, especially in humanics security, entertaining security, user authentication, and anti-malware techniques. He is currently serving as Chief Examiner of IPSJ (Information Processing Society of Japan) Special Interest Group on Computer Security.

Kunihiro Yamada received the M.E. degree from the Ritsumeikan University in 1973 and received the Ph.D. degree industrial science and engineering from Shizuoka University in 2002.He joined the Mitsubishi Electric Corp from 1973 to 2003,and joined the Renesas Solutions Corp as the board director to 2005. He joined the professional graduate school of embedded technology of Tokai University to 2013 as a professor. He was a visiting professor at the Department of System Design, Tokyo Metropolitan University , and adviser at the MegaChips Corporation since 2005.His major research are improvement by structural reforms of microprocessor and the mutual complement networks by wired and the wireless. He is one of the members of IPSJ Japan.

Tadanori Mizuno received the B.E. degree in Industrial Engineering from the Nagoya Institute of Technology in 1968 and received the Ph.D. degree in Computer Science from Kyushu University, Japan, in 1987. In 1968, he joined Mitsubishi Electric Corp. From 1993 to 2011, he had been a Professor of Shizuoka University, Japan. Since 2011, he is a Professor of the Aichi Institute of Technology, Japan. His research interests include mobile computing, distributed computing, computer networks, broadcast communication and computing, and protocol engineering. He is a member of Information Processing Society of Japan, the Institute of Electronics, Information and Communication Engineers, the IEEE Computer Society, ACM and Informatics Society.